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93201Q

932012

NEW ZEALAND QUALIFICATIONS AUTHORITY
MANA TOHU MĀTAURANGA O AOTEAROA

Scholarship 2013
Statistics

9.30 am Monday 11 November 2013
Time allowed: Three hours
Total marks: 40

QUESTION BOOKLET

There are FIVE questions in this booklet. Answer ALL questions.

Pull out Formulae and Tables Booklet S–STATF from the centre of this booklet.

Write your answers in Answer Booklet 93201A.

Show ALL working. Start your answer to each question on a new page. Carefully number each question.

Check that this booklet has pages 2–11 in the correct order and that none of these pages is blank.

YOU MAY KEEP THIS BOOKLET AT THE END OF THE EXAMINATION.

You have three hours to complete this examination.

QUESTION ONE (8 marks)

Different breeds of cow are known to have different characteristics in the milk they produce. Jersey and Ayrshire are two common breeds in New Zealand. During the 2012–2013 milking year approximately 350 000 Jersey cows and 18 000 Ayrshire cows were tested four times.

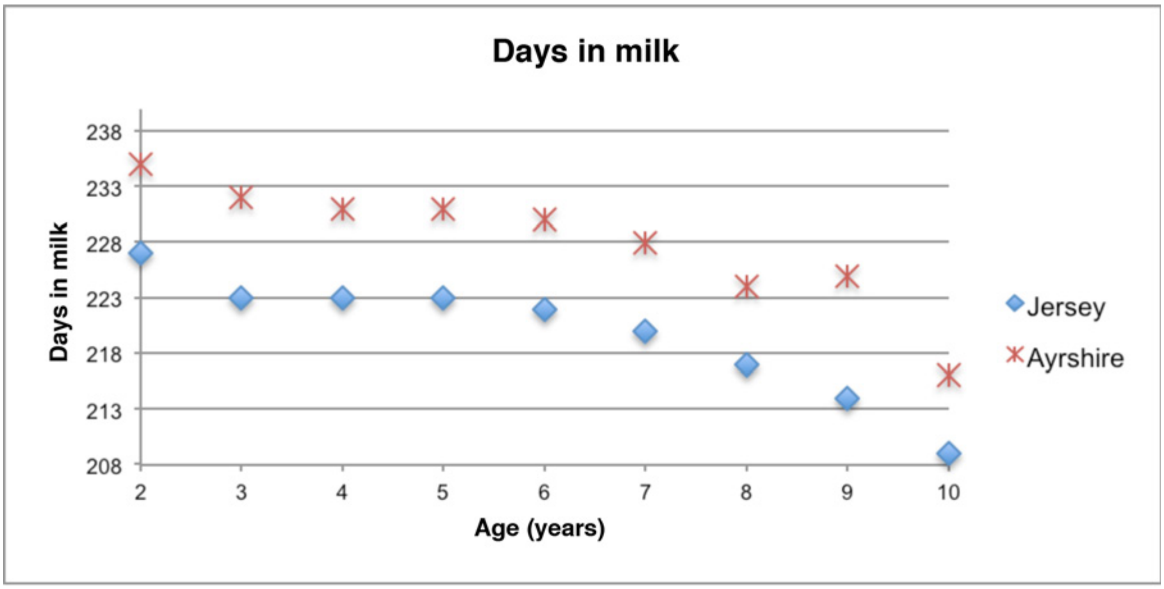
The age and breed of each cow was recorded and, on the four test days, the amounts of milk produced and the results of a chemical analysis of the milk were recorded. The analysis included the amounts of fat and protein in the milk.

From the data, the annual milk production of each cow was estimated.

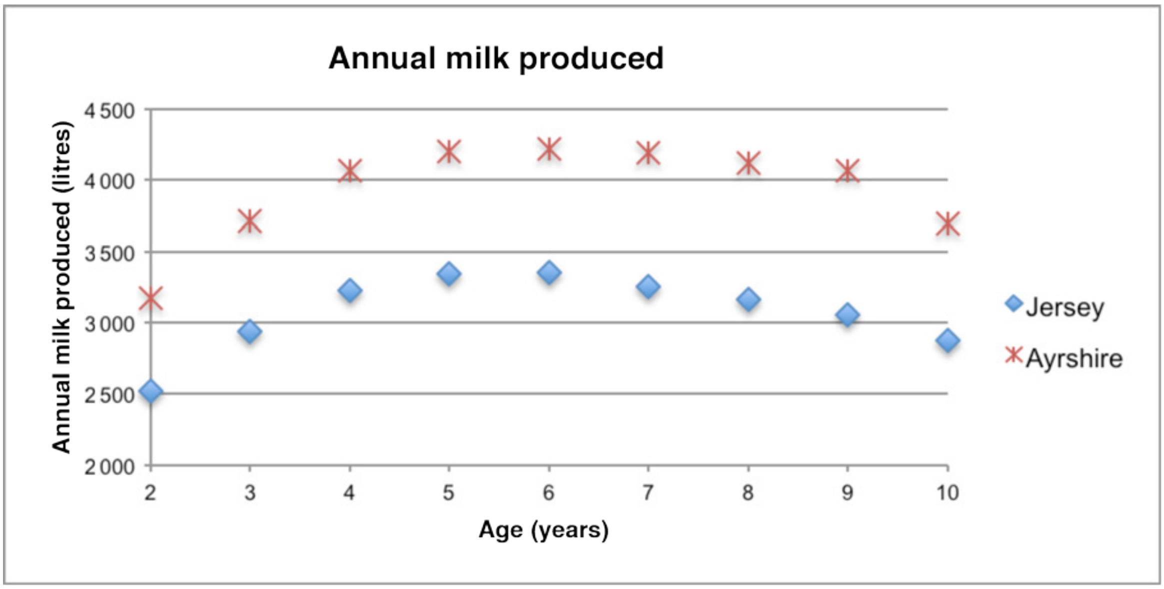
(a) For each breed and age, the following were calculated:

Days in milk	The average number of days in the milking year that a cow produces milk.
Annual milk produced	The average amount of milk produced in the milking year per cow, in litres.
Milk fat percentage	The average percentage of fat in milk.

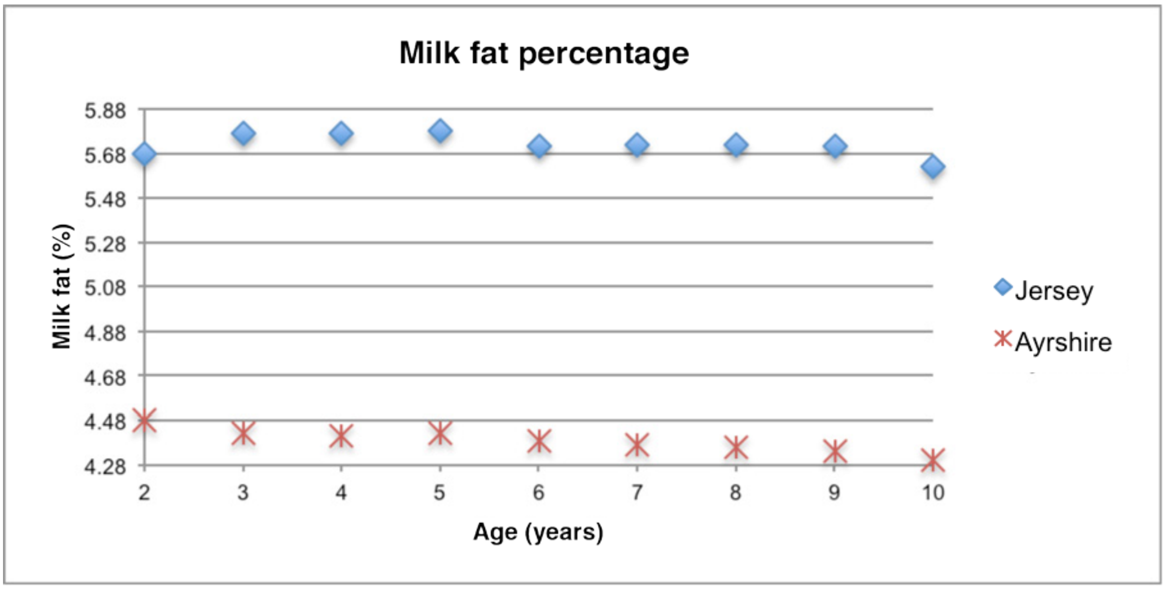
Graphs 1 to 3 show this information.



Graph 1



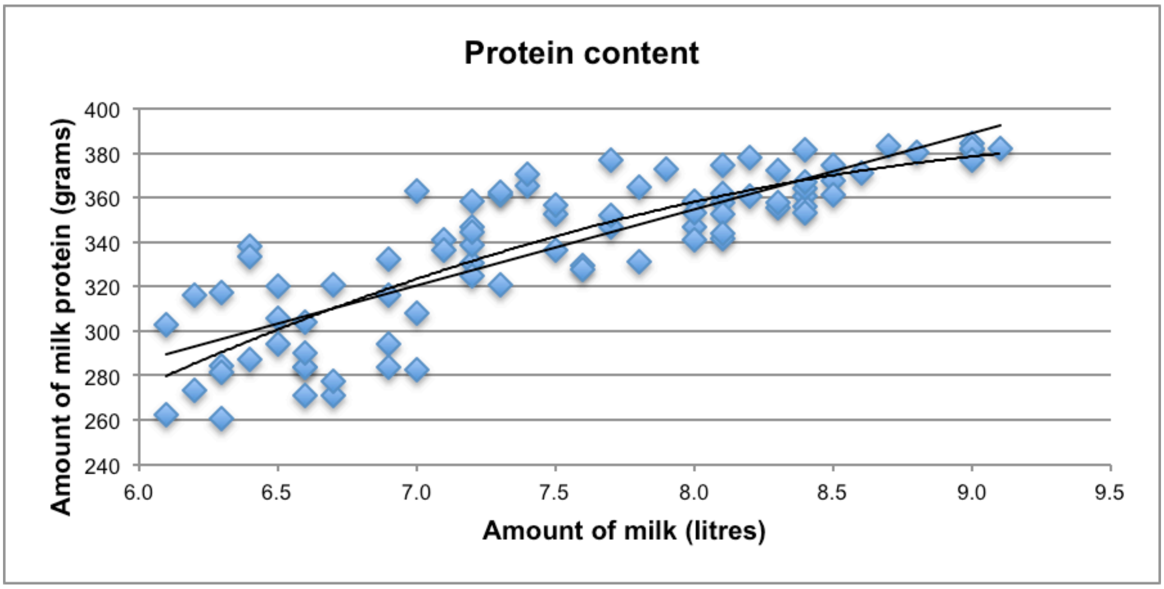
Graph 2



Graph 3

Write a half page report that compares and contrasts milk production from each breed.

(b) On one particular day, a herd of 85 Jersey cows was milked. The milk was analysed for its protein content, and the data graphed in Graph 4.



Graph 4

(i) Discuss the relationship between the amount of milk produced, and the amount of milk protein produced, for this herd of Jersey cows.

(ii) Two possible trend models were added to the plot. The equations of these trend models are:

$$y = -7.2x^2 + 142x - 320$$

$$y = 34.3x + 80$$

where x is the amount of milk produced, in litres, and y is the amount of milk protein produced, in grams.

1. By choosing an appropriate model in each case, predict the amount of milk protein for a cow that produced 7.5 litres of milk, and for another cow that produced 9.2 litres of milk.

   In each case show how you arrived at your prediction.

2. Discuss the precision of your predictions.

3. Identify two factors that would need to be considered before these predictions could be applied to the whole population of Jersey cows in New Zealand.

   Discuss why information about these two factors could be useful.

QUESTION TWO (8 marks)

(a) It is known that the presence of internal parasites adversely affects the health of a cow and the amount of milk it produces. An experiment to test the effectiveness of a new anti-parasite product compared to a currently used product was carried out on a small herd of 54 cows. The cows were assigned randomly to two equally sized groups; group A and group B. Cows in group A were treated with the currently used product, and cows in group B were treated with the new product. The cows were kept as one herd and had the same grazing conditions. Six weeks after receiving the treatment the total amount of milk each cow produced over a period of one week was recorded (in litres).

Figure 1 shows a dot plot of the data. The vertical lines show the group means of 90.49 litres for group A and 92.18 litres for group B. The difference in the group means of 1.69 litres is also displayed.

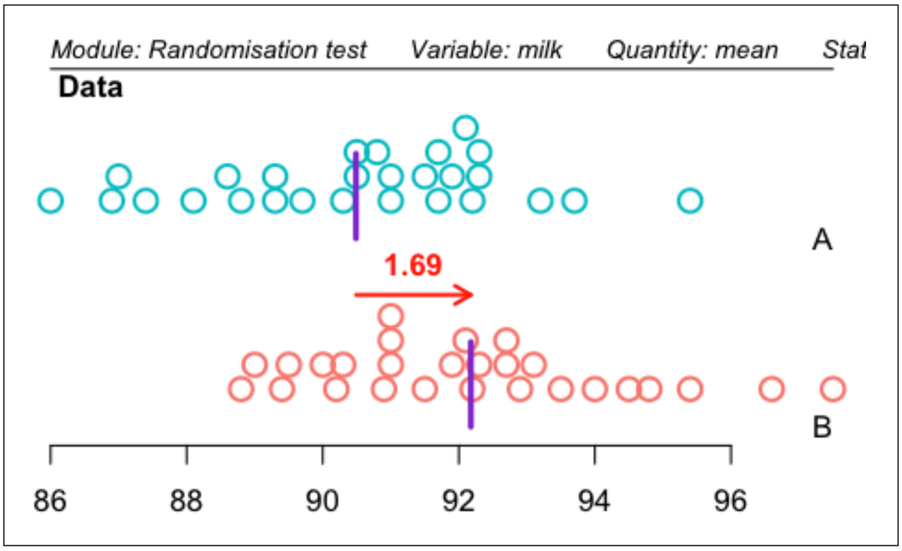


Figure 1

A randomisation test was carried out on the data and the resulting test output is shown in Figure 2. The tail proportion produced by the test is 0.004.

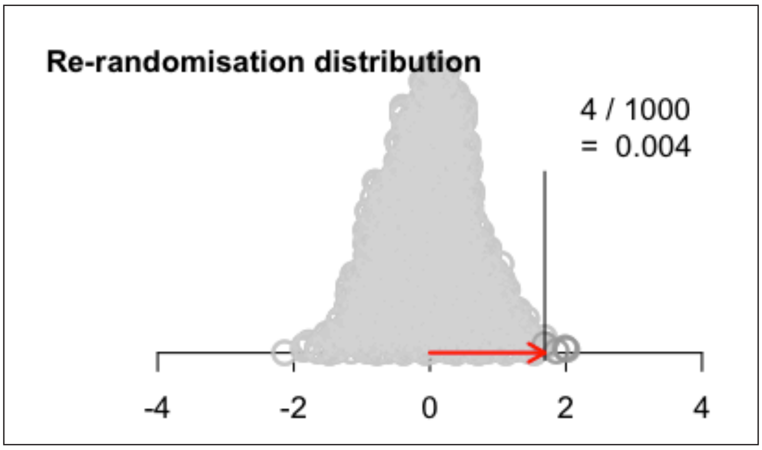


Figure 2

(i) What can be concluded from this output?

Justify your answer.

(ii) Identify and discuss two experimental design factors that need to be considered before it could be claimed that the new treatment was a superior product to the current one.

(b) The live weight of cows is important information for a farmer. Large cows need a lot more feed for body maintenance than small cows, and thus need more grass.

Jersey cows are a small breed and are considered to be easy to handle. They are calm and docile animals, but tend to require more attentive management in cold weather than other dairy breeds, due to their smaller body mass.

Ayrshire cows are rugged animals, ideal for pasture-based milk production. They are more adapted to rugged terrain and adverse foraging conditions than most major dairy breeds are. They have very few foot and leg problems, and have a high resistance to disease.

Figure 3 shows dot plots and box-and-whisker plots of the weights (in kilograms) of random samples of Jersey and Ayrshire cows.

Table 1 gives summary statistics for these weights.

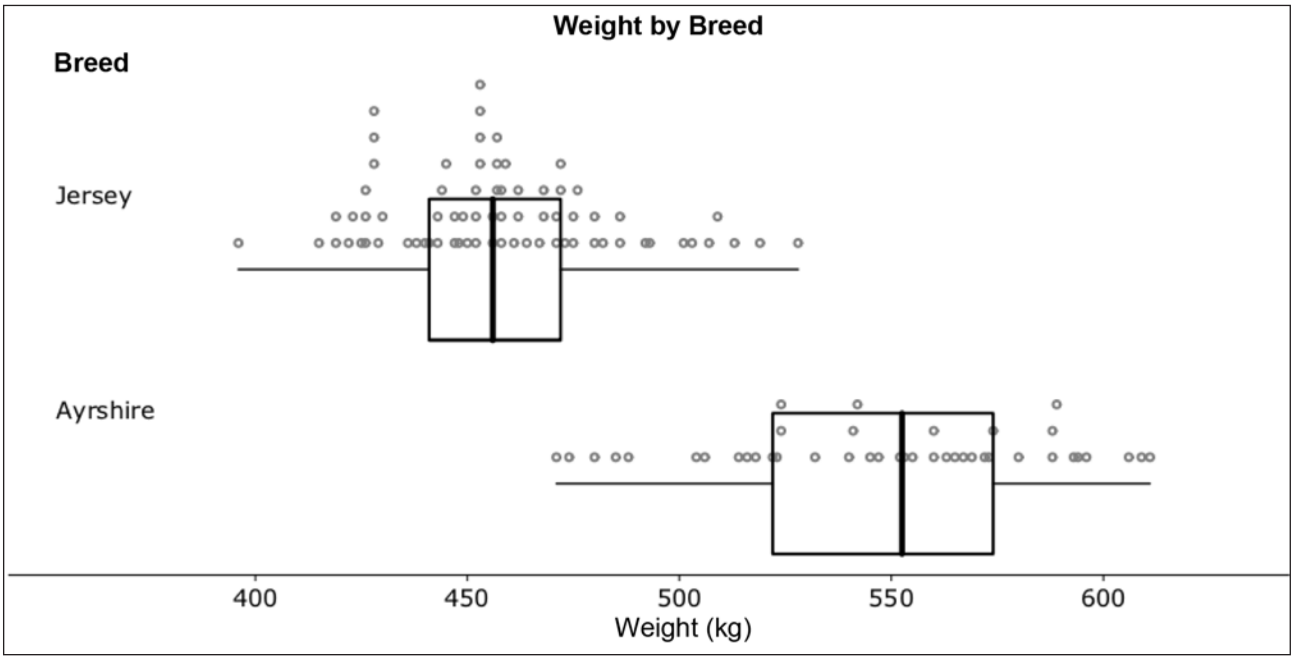


Figure 3

Breed	Weights in kg						Standard deviation	Sample size
	Minimum	Lower quartile	Median	Mean	Upper quartile	Maximum		
Jersey	396	441	456	457.6	472	528	26.6	73
Ayrshire	471	522	552	547.9	573	611	38.3	42

Table 1

Bootstrap confidence intervals for the mean weight of cows from each breed were:

Jersey (451.5 kg, 464.1 kg)

Ayrshire (535.6 kg, 559.0 kg)

(i) Write a short paragraph comparing the weights of Jersey and Ayrshire cows.

(ii) A cow needs to eat about 3% of its own body weight in dry matter each day.

Taking into account weight variation, estimate the difference in mean daily dry matter food requirements between Jersey and Ayrshire cows.

Comment on the validity of your answer.

QUESTION THREE (8 marks)

(a) A yoghurt manufacturer has developed a new style of yoghurt using a different formula. The manufacturer wants to know if customers will be able to distinguish the new yoghurt from the currently produced yoghurt. A triangle test is used in which a taster tastes three samples of yoghurt; two are samples of the current yoghurt, and one is the new yoghurt. The yoghurts are presented in a random order. The taster is asked to identify the sample of the new yoghurt. Out of a panel of 15 tasters, 11 correctly identified the sample of the new yoghurt.

Is this sufficient evidence to conclude that the new yoghurt is distinguishable?

In your answer justify the choice of any probability model used.

(b) Ice-cream tubs are required to have a weight between 500 g and 520 g in order to be considered acceptable. As part of a quality control process a large random sample of tubs was taken which had a mean of 513 g with a standard deviation of 3.5 g.

Assuming a normal distribution model, determine whether current production is meeting acceptable requirements.

Justify your answer.

(c) A dairy factory produces bags of milk powder which are filled by an automated machine. On average, the filling machine under-fills six bags per hour. Some factory workers, called operators, are used to top up any under-filled bags. Each operator can, in addition to their regular work, top up at most two bags every hour.

The company wants to make sure that, in the long run, no bags are left under-filled at least 90% of the time.

Calculate the smallest number of operators the company requires.

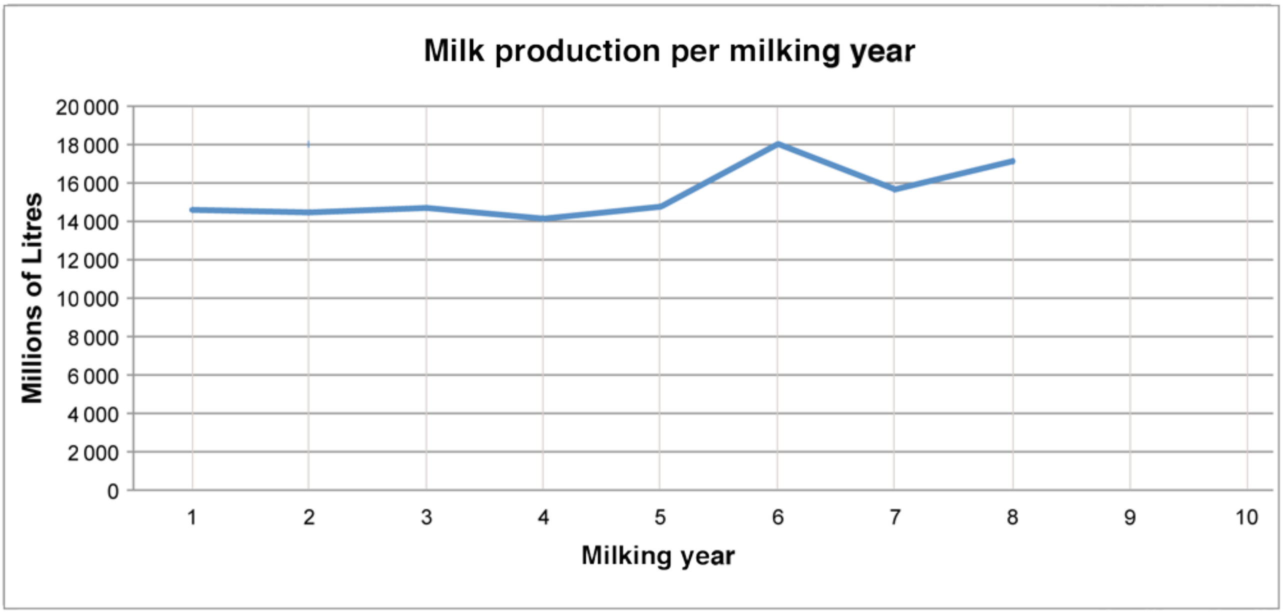
In your answer justify the choice of any probability model used.

QUESTION FOUR (8 marks)

(a) Graphs 5 and 6 show milk production, in millions of litres, over the period from the start of the 2003 – 2004 milking year to the end of the 2010 – 2011 milking year. Each milking year consists of four seasons in the order: Winter, Spring, Summer, and Autumn.

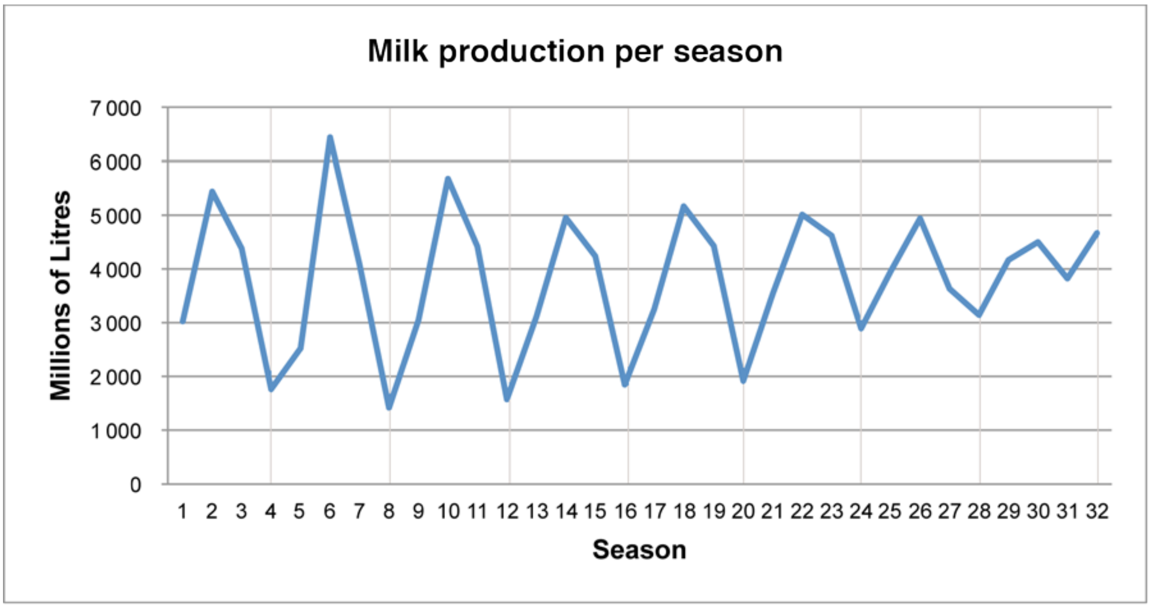
In Graph 5, Year 1 corresponds to the year from Winter 2003 to Autumn 2004, Year 2 the year from Winter 2004 to Autumn 2005, and so on.

Graph 5 shows milk production for each milking year from the 2003 – 2004 (milking year 1) to the 2010 – 2011 milking year (milking year 8).



Graph 5

Graph 6 shows milk production for each season from Winter 2003 (Season = 1) to Autumn 2011 (Season = 32).



Graph 6

Using these graphs, write a paragraph describing milk production over these years.

(b) Tables 2 and 3 give the milk production for each season in milking year 8, and the total milk production for milking years 5 to 8 respectively.

Milking Year 8	Milk production (millions of litres)
Winter	4 161
Spring	4 502
Summer	3 815
Autumn	4 661

Table 2

Milking year	Milk production (millions of litres)
5	14 746
6	18 044
7	15 670
8	17 139

Table 3

By using extrapolation or otherwise, find a forecast for milk production for Autumn 2013 (autumn of milking year 10).

Comment on the validity of your forecast.

(c) Table 4 gives milk fat prices and dairy land sale values alongside the Consumer Price Index (CPI) for every four years over the years from 1998 to 2010. The CPI is a measure of inflation.

Year	Milk fat price ($ per kg)	Dairy land sale value ($ per hectare)	CPI
1998	32.00	11 076	835
2002	45.60	14 658	958
2006	65.40	29 281	1 087
2010	65.80	30 530	1 191

Table 4

Discuss the changes in milk fat prices and dairy land sale values in terms of real dollars (ie, adjusting for inflation).

QUESTION FIVE (8 marks)

Read the following report extract and answer the question at the end:

The Impact of Lameness on the Milk Yield of Dairy Cows

Introduction

This report investigates the impact of lameness in cows on milk yield. The dataset includes approximately 8 000 test day milk yields from 900 cows on five farms in Gloucester UK collected over 18 months from 1997 to 1999. Factors affecting milk yield included: farm of origin, stage of lactation (time since giving birth, measured in periods of three months), parity (the number of times a cow has given birth), and whether a cow ever became lame or not. Cows are assumed to produce milk for 305 days after giving birth; called 305 days in milk (DIM).

Results

Over 70% of cows became lame at least once. The four most frequent diagnoses of lameness were sole ulcer, white line disease, inter-digital necrobacillosis and digital dermatitis. These had an incidence of 9

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Graph 7

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Graph 8

Predicted milk yields (lame in 2nd month of milk). The arrow indicates the time of lameness diagnosis. The dotted line indicates the potential milk production of the lame cow.

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Table 5

to 11 cases per 100 cows per year. The incidence of first episode of lameness peaked three months after calving (refer Graph 7). High-yielding cows are more likely to become lame. These high-yielding lame cows produced a mean of 1.12 (±0.34) kg/day more milk than cows that were never lame: that is a mean of 342 extra kilograms of milk over 305 DIM (95% CI 135 to 549 kg). Lame cows had a reduced milk yield from up to four months before a case of lameness was diagnosed and treated, and for five months after treatment (refer Table 5 with s.e. denoting standard error). The total mean estimated reduction in milk yield for a cow lame in the fifth month of lactation onwards was 357 kg (95% CI 163 to 552) per 305 day lactation (refer Table 5). Graph 8 illustrates the impact on milk yield for a cow lame two months after calving, together with the estimated milk yield had the cow not become lame, compared with the mean yield of a cow that was not lame.

The estimates from this report indicate that lame cows fail to produce an average of approximately 350 kg of milk, therefore this advantage of higher yield is lost. The mean range of milk lost per affected cow was between 160 and 550 kg. The wide range in predicted loss occurred because all causes of lameness were included in the analysis, and in some cases some causes may have impacted on milk yield more than others. Variability in the cause of lameness, and the time of lameness during lactation, will have also led to variability in the mean estimate of milk lost by month (refer Table 5).

Recent work indicates that farmers underestimate the prevalence of lameness in their cows considerably; in a study of 53 herds, the mean estimate of lame cows was 5% by farmers versus 25% by the researcher. Another likely possibility for the delay in treatment may be that these cows were not lame in early lactation but eventually became lame. This highlights the important issue of diagnosis. There is clearly a need to improve the detection of clinical lameness and to remove the subjective assessment of the human observer, whether farmer or veterinarian or agricultural consultant.

Conclusions
We conclude that some of the potential of high yielding cows in a herd may be lost if they become lame. In lame cows, milk yield was reduced from up to four months before a case of lameness was diagnosed and treated, and for the five months after treatment. The total mean estimated reduction in milk yield per 305 day lactation was approximately 360 kg, with a range 160 to 550 kg. **We conclude that lameness has a significant impact on milk production.** This is important information for assessing the economic impact of lameness and its impact on cow health. It adds weight to the importance of early identification of lameness and the urgency of improving techniques of diagnosis.

Source (adapted): *Journal of Dairy Science* Vol. 85, No. 9, 2002.

Identify three distinct features that relate to lameness and milk production of cows.

For each feature:

- describe the feature
- discuss the evidence in the report that supports the feature
- discuss how the feature relates to the **bolded** conclusion.

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